

Pre-Calc 12
Final Exam Review

Name: _____

Ch 1 Transformations

1. If $f(x) = 5x - 1$, find the equation of a) $f^{-1}(x)$ and b) $f(x)$ translated 4 units to the right.

2. If (a, b) is a point on the graph of $y = f(x)$, what point must be on the following?

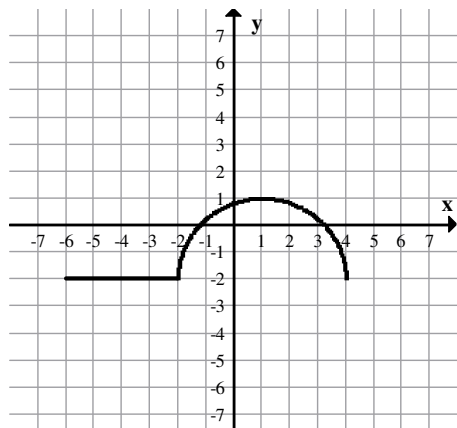
a) $f(2x+10)$

b) $2f(x+3)+2$

c) $-\frac{1}{2}f\left(\frac{1}{3}x-2\right)+7$

3. If $f(x) = 3x + 2$, find the equation of a) its inverse b) a reflection in the x -axis.

4. The graph of $y = f(x)$ is shown below. Sketch the graphs of:



a) $f\left(\frac{1}{2}x\right)$

b) $-f(2(x+1))$

c) $-f\left(\frac{1}{2}x-3\right)+1$

d) $\frac{1}{2}f(-x+2)-1$

5. Given $f(x) = x^2 - 4$ and $g(x) = 2x + 3$, find:

a) $g(x) - f(x)$

b) $g(f(-2))$

c) $g(f(x))$

Ch 2 Polynomials

1. What is the remainder when $x^{12} - 2x^7 + 6x^2 - 4$ is divided by $x + 1$?

2. Divide $x^3 - 2x + 3$ by $x + 2$.

3. Solve by factoring:

a) $x^3 - 7x - 6 = 0$

b) $3x^3 - 16x^2 - 13x + 6 = 0$

4. Sketch the graph of: $y = x^3 - 7x - 6$.

5. Find the equation of a polynomial that has zeroes at $-2, 1, 4$ and has a y -intercept of -16 .

6. Solve the following inequalities:

a) $2(x+4)^2(x-3)(x-5) \geq 0$

b) $-(x-4)(x-1)(x+2) < 0$

- A rectangular prism has dimensions 10cm by 10cm by 5cm. When each dimension is increased by the same amount, the new volume is 1008cm^3 . What are the dimensions of the new prism?
- A cereal box is a rectangular prism with a volume of 2500cm^3 . It is 4 times as wide as it is deep and 5cm taller than it is wide. What are the dimensions of the box?

Ch 3 – Radical and Rational Functions

- Determine the domain and range of each:

a) $y = \sqrt{2(x+3)}$

b) $f(x) = -2\sqrt{3x+1} - 1$

- Solve:

a) $x = \sqrt{x+10} + 2$

b) $\sqrt{4x+5} - \sqrt{2x-1} = 2$

- Sketch: a) $y = 4x$ and b) $y = \sqrt{4x}$

- Given $y = \frac{x^2 - 2x - 8}{x^2 + x - 2}$, find the equations of all asymptotes, the coordinates of any holes, the x and y-intercepts, and the domain. Sketch a graph.

Ch 4 – Logarithms

- Write as a single log:

a) $3 + 5\log_2 a - 4\log_2 b - \frac{1}{2}\log_2 c$

b) $\log_{25} 75 + \log_5 3 - \log_{25} 5$

c) $\log_2(2x) + \log_4(3x) - \log_{16}(x)$

- Find the exact value of:

a) $\log_4 16 - \log_3 \left(\frac{1}{27} \right)$

b) $19^{\log_{19} 17}$

c) $\log_5 \sqrt{175} - \log_{25} 7$

- Simplify: $49^{2\log_7(3x)}$

- If $\log_7 x = m$ and $\log_7 y = n$, rewrite the following in terms of m and n :

a) $\log_7(49x^2y)$

b) $\log_{49}(7xy^3)$

- Solve for x algebraically:

a) $8 \times 4^{x+2} = 64^{1-2x}$

b) $3^{x+2} = 5^{2x-3}$

c) $\log_2(x+2) = 3 - \log_2 x$

d) $(\log_5 8)(\log_x 5) = 3$

- Balonium decays from 5000 atoms to 10 atoms in 12 seconds. What is the half-life of Balonium? (Answer to two decimal places)
- An earthquake off the coast of Tofino was measured as 5.2 on the Richter scale. Experts expect the next major earthquake to be about 40 000 times stronger. What will the Richter scale number of this earthquake be?

8. Milk has a pH of 6.6 and apples are about 800 times more acidic. Find the pH of apples.
9. Sketch: $y = 2^{x-3} - 4$. Determine the domain, range and equations of any asymptotes.
10. Sketch: $y = \log_3(x+2)$. Determine the domain, range and equations of any asymptotes.
11. What is the domain of: $\log_{x+3}(7-x)$?

Ch 5 – Trig Part 1

1. Convert into degrees: a) $\frac{7\pi}{6}$ b) -1.46
2. Convert into radians: a) 280° b) -520°
3. A pendulum 30 cm in length swings through an arc length of 20 cm. What is the angle in degrees?
4. For each, sketch the angle and determine: i) a coterminal angle and ii) the reference angle
 - a) 290° b) $\frac{5\pi}{7}$ c) $-\frac{2\pi}{3}$
5. Given that $(-6,8)$ is on the terminal arm of angle θ in standard position, find the exact values of the following trig ratios.
 - a) $\sin \theta$ b) $\sec \theta$ c) $\cot \theta$
6. Given $\sin \theta = -\frac{5}{13}$ and θ is in quadrant III, find the exact values of:
 - a) $\cos \theta$ b) $\csc \theta$ c) $\tan \theta$
7. Given $\cos \theta = \frac{a-1}{a}$ where $a > 0$ and $\csc \theta < 0$, find the exact value of $\sin \theta$.
8. Find the exact values of:
 - a) $\cos\left(-\frac{7\pi}{3}\right)$ b) $\sec\left(\frac{5\pi}{6}\right)$ c) $\sin\left(-\frac{3\pi}{4}\right)$
9. Given $\sin \theta = -\frac{1}{\sqrt{2}}$ and $\sec \theta < 0$, what is the exact value of angle θ if $0 \leq \theta < 2\pi$?
10. Determine the amplitude, period, phase shift, vertical displacement and the range of:
 - a) $y = -3\cos \pi x - 5$ b) $y = 2\cos(2x - 6\pi) + 1$

11. Determine the period and equation of the asymptotes of:

a) $y = 3 \tan 5x + 1$

b) $y = -\tan\left(\frac{1}{3}x\right)$

12. Write the equation of a sine curve that has a minimum at $(3, -1)$ and the next maximum at $(5, 7)$.

13. On a typical day at an ocean port, the water has a minimum depth of 12 m at 2:00am. The maximum depth of 20 m occurs 6 hours later. Assume that the relationship between the depth of water and time is a sinusoidal function. Write an equation for the depth of water at any time, t hours.

Ch 6 – Trig Part 2

1. Simplify the following expressions:

a) $\frac{1}{\csc^2 x - 1}$

b) $2 \cos(2x) \cos(3x) + 2 \sin(2x) \sin(3x)$

c) $\csc(2A) \sin A$

d) $4 - 8 \cos^2(6x)$

2. If $\cot \theta = -\frac{12}{5}$ and $\cos \theta < 0$, find the exact value of $\sin(\theta + \pi)$.

3. Solve: $2 \sin x + \sqrt{3} = 0$ if a) $0 \leq x < 2\pi$ and b) over all real numbers.

4. Solve: $\cot x = \sqrt{3}$ if a) $0 \leq x < 2\pi$ and b) over all real numbers.

5. Solve: $\cos 2x = -\frac{\sqrt{3}}{2}$ if a) $0 \leq x < 2\pi$ and b) over all real numbers.

6. Solve algebraically with exact values: $2 \sin^2 x + 11 \sin x + 5 = 0$ over all real numbers.

7. Prove:

a) $\frac{\tan \theta}{\sec \theta} + \frac{1}{\sec \theta} = \frac{\cos 2\theta}{\cos \theta - \sin \theta}$

b) $\frac{\csc \theta + 1}{\cos \theta} = \frac{\cot \theta}{1 - \sin \theta}$

8. Determine the restrictions on x : $\frac{\tan x}{\sec x - 1}$ if $0 \leq x < 2\pi$.

Ch 7 – Combinatorics

1. A couple is planning an evening out. They have a choice of 4 restaurants for dinner, 6 movies following dinner and 4 coffee shops for after the movie. How many different ways can they plan the evening if they choose one of each?

2. How many different ways are there to arrange the letters in TSAWWASSEN if:

- there are no restrictions
- if the first letter must be an S
- if the As must be together

3. The winner of a lottery chooses 5 vehicles from a warehouse that contains 13 different cars, 9 different trucks and 6 different motorcycles. How many different 5 choices of vehicles are possible if there must be at least one car?
4. Codes with 5 digits are made from the digits 1, 2, 3, 4, 5, 6, 7, 8.
 - a) If repetitions are allowed, how many different 5-digit codes are there?
 - b) If repetitions are not allowed and each code must contain 3 odd digits followed by 2 even digits, how many different codes can be made?
5. A class of 26 students is made up of 16 boys and 10 girls. From this class, a group of 5 students is chosen.
 - a) Determine the number of different groups of 5 that can be formed if there must be 2 girls and 3 boys in the group.
 - b) Determine the number of different groups of 5 that can be formed if there must be at most 1 boy in each group.
6. In a standard deck of 52 cards, how many different 5-card hands contain at least 3 face cards?
7. Solve algebraically:
 - a) ${}_n P_2 = 90$
 - b) $\frac{n!}{(n-2)!3!} = 5$
8. Simplify: $\frac{(n-2)!(n+1)!}{(n!)^2}$
9. Determine the 7th term in the expansion of $(2x - y)^{11}$.
10. In the expansion of $(a^2 - b)^4$, determine the middle term.

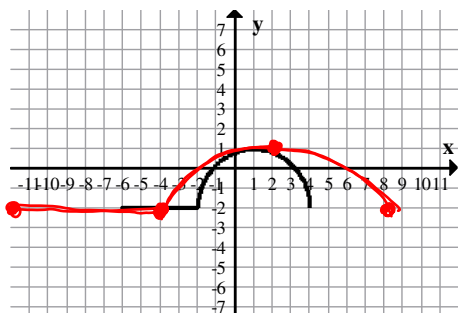
Answer Key:

Ch 1

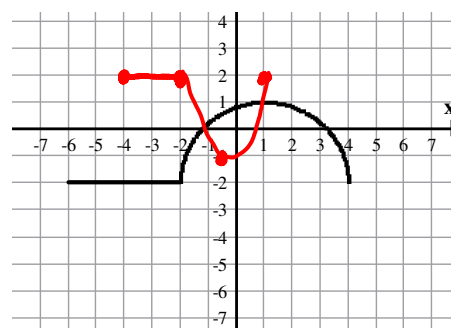
1. a) $y = \frac{x+1}{5}$ b) $y = 5x - 21$ 2. a) $\left(\frac{1}{2}a - 5, b\right)$ b) $(a - 3, 2b + 2)$ c) $\left(3a + 6, -\frac{1}{2}b + 7\right)$

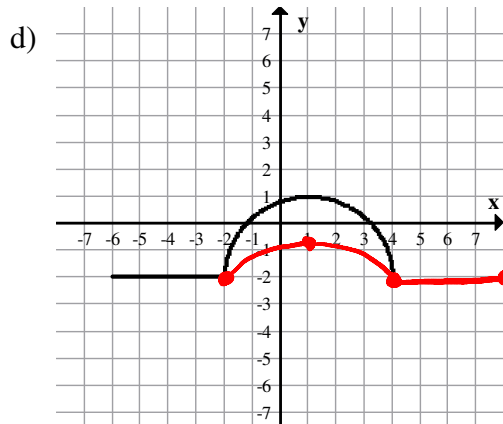
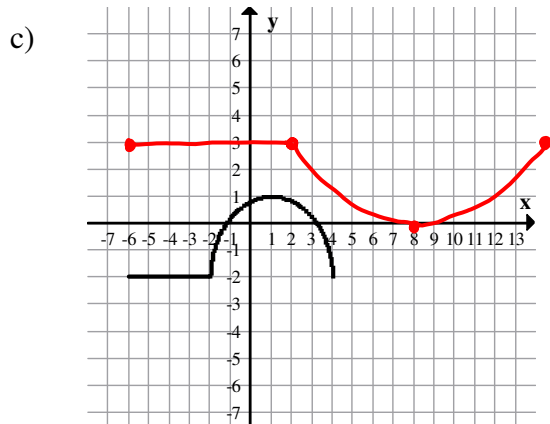
3. a) $y = \frac{x-2}{3}$ b) $y = -3x - 2$

4. a)



b)

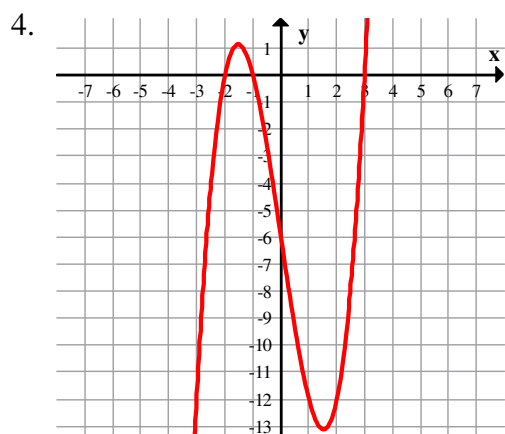




5. a) $-x^2 + 2x + 7$ b) 3 c) $2x^2 - 5$

Ch 2

1. 5 2. $(x^2 - 2x + 2) - \frac{1}{x+2}$ 3. a) $x = -1, x = -2, x = 3$ b) $x = -1, x = \frac{1}{3}, x = 6$



5. $y = -2(x+2)(x-1)(x-4)$

6. a) $x \leq 3, x \geq 5$ or $(-\infty, 3], [5, \infty)$

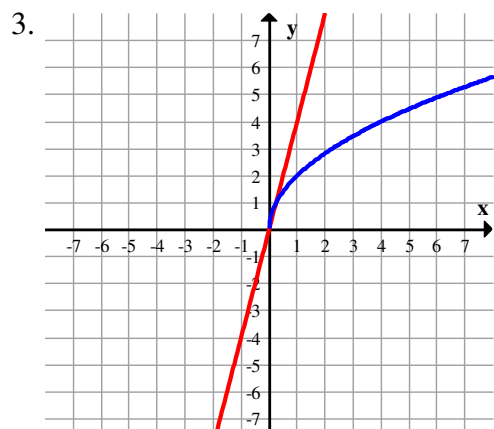
b) $-2 < x < 1, x > 4$ or $(-2, 1), (4, \infty)$

7. $12\text{cm} \times 12\text{cm} \times 7\text{cm}$

8. $5\text{cm} \times 20\text{cm} \times 25\text{cm}$

Ch 3

1. a) $x \geq 3, y \geq 0$ b) $x \geq 3, y \leq -1$ 2. a) $x = 6$ b) $x = 5, x = 1$



4. asymptotes: $x = 1, y = 1$ hole: $(-2, 2)$

intercepts: $(4, 0)$ and $(4, 0)$

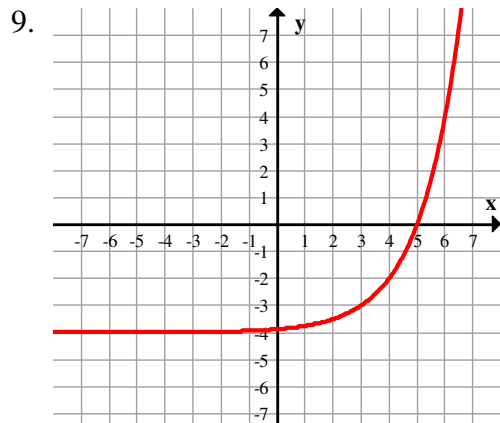
domain: $x \neq -2, 1$ or $(-\infty, -2), (-2, 1), (1, \infty)$

Ch 4

1. a) $\log_2 \left(\frac{8a^5}{b^4 c^{\frac{1}{2}}} \right)$ b) $\log_{25} 135$ c) $\log_{16} (144x^5) = \log_2 \left(2\sqrt{3}x^{\frac{5}{4}} \right)$ 2. a) 5 b) 17 c) 1

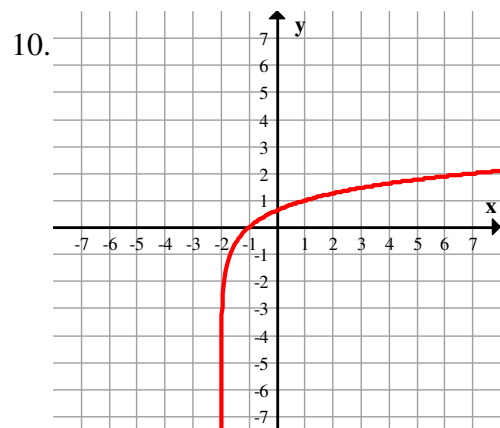
3. $81x^4$ 4. a) $2+2m+n$ b) $\frac{1}{2}+\frac{m}{2}+\frac{3n}{2}$ 5. a) $x=-\frac{1}{14}$ b) $x=3.31$ c) $x=2$ d) $x=2$

6. 1.34 seconds 7. 9.8 8. 3.10



Domain and Range: $x \in \mathbb{R}, y > -4$

Asymptote: $y = -4$



Domain and Range: $x > -2, y \in \mathbb{R}$

Asymptote: $x = -2$

11. $-3 < x < 7$

Ch 5

1. a) 210° b) -83.65° 2. a) $\frac{14\pi}{9}$ b) $-\frac{26\pi}{9}$ 3. 38.2° 4. a) $\mathbb{R}: 70^\circ$ b) $\mathbb{R}: \frac{2\pi}{7}$ c) $\mathbb{R}: \frac{\pi}{3}$

5. a) $\frac{4}{5}$ b) $-\frac{4}{3}$ 6. a) $-\frac{12}{13}$ b) $-\frac{13}{5}$ c) $\frac{5}{12}$ 7. $\frac{-\sqrt{2a-1}}{a}$ 8. a) $\frac{1}{2}$ b) $-\frac{2}{\sqrt{3}}$ c) $-\frac{1}{\sqrt{2}}$

9. $\frac{5\pi}{4}$ 10. a) amp: 3, period: 2, p.s: 0, vd: down 5, range: $-8 \leq y \leq -2$

b) amp: 2, period: π , p.s: right 3π , vd: up 1, range: $-1 \leq y \leq 3$

11. a) period: $\frac{\pi}{5}$, asymp: $x = \frac{n\pi}{10}$, n is odd b) period: 3π , asymp: $x = 3n\pi$, n is odd

12. $y = 4 \sin \frac{\pi}{2}x + 3$, $y = 4 \cos \frac{\pi}{2}(x-1) + 3$ 13. $y = 4 \sin \frac{\pi}{6}(x-5) + 16$, $y = 4 \cos \frac{\pi}{6}(x+4) + 16$

Ch 6

1. a) $\tan^2 x$ b) $2 \cos(-x)$ c) $\frac{\sec A}{2}$ d) $-4 \cos 12x$ 2. $-\frac{5}{13}$ 3. a) $x = \frac{4\pi}{3}, \frac{5\pi}{3}$

b) $x = \frac{4\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi, n \in \mathbb{I}$ 4. a) $x = \frac{\pi}{6}, \frac{7\pi}{6}$ b) $x = \frac{\pi}{6} + n\pi, n \in \mathbb{I}$

5. a) $x = \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}$ b) $x = \frac{5\pi}{12} + n\pi, \frac{7\pi}{12} + n\pi, n \in I$ 6. $x = \frac{7\pi}{6} + 2n\pi, \frac{11\pi}{6} + 2n\pi, n \in I$

7. a)
$$\frac{\frac{\sin \theta}{\cos \theta} \times \cos \theta + \cos \theta}{\sin \theta + \cos \theta} \left| \begin{array}{l} \frac{\cos^2 \theta - \sin^2 \theta}{\cos \theta - \sin \theta} \\ \frac{(\cos \theta + \sin \theta)(\cos \theta - \sin \theta)}{(\cos \theta - \sin \theta)} \\ \cos \theta + \sin \theta \end{array} \right.$$

b)
$$\frac{(\csc \theta + 1) \times \frac{(1 - \sin \theta)}{(1 - \sin \theta)}}{\cos \theta} \left| \begin{array}{l} \frac{\csc \theta - 1 + 1 - \sin \theta}{\cos \theta (1 - \sin \theta)} \\ \frac{\frac{1}{\sin \theta} - \sin \theta}{\cos \theta (1 - \sin \theta)} \\ \frac{1 - \sin^2 \theta}{\sin \theta} \\ \frac{\sin \theta}{\cos \theta (1 - \sin \theta)} \\ \frac{\cos^2 \theta}{\sin \theta} \\ \frac{\cot \theta}{1 - \sin \theta} \end{array} \right.$$

8. $x \neq 0, \frac{\pi}{2}, \frac{3\pi}{2}$

Ch 7

1. 96 2. a) 151 200 b) 45 360 c) 30 240
 3. 95 277 4. a) 32 768 b) 288 5. a) 25 200 b) 3 612
 6. 192 192 7. a) $n = 10$ b) $n = 6$ 8. $\frac{n+1}{n(n-1)}$
 9. $14784x^5y^6$ 10. $6a^4b^2$