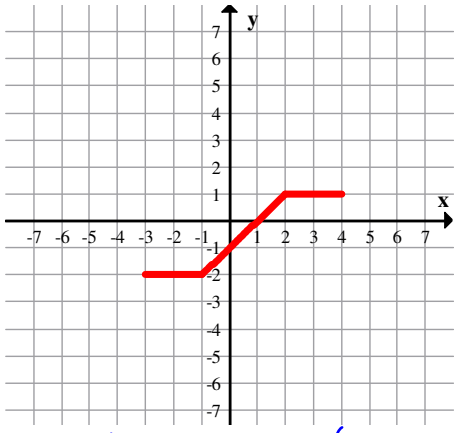
Pre-Calc 12
Midterm 1 Review **Name:** KEY

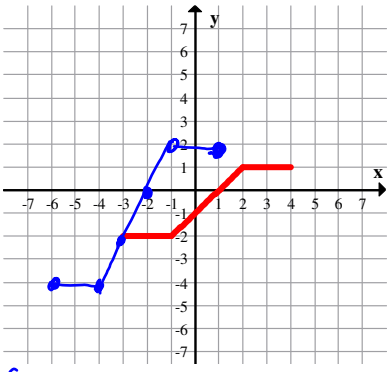
Ch 1 Transformations

1. Given $y = f(x)$, sketch a graph of the following:
   a. $y = 2f(x+3)$
   b. $y = f(2x-4)-1$
   c. $y = -f\left(\frac{1}{3}x\right)+2$
   d. $y = f^{-1}(x)$
   e. $y = f(-x-1)$

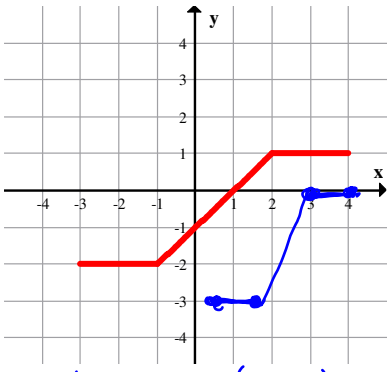


$f(2(x-2))-1 \rightarrow \left(\frac{1}{2}x+2, y-1\right)$

a.

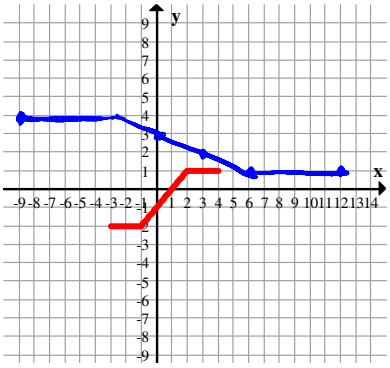


b.

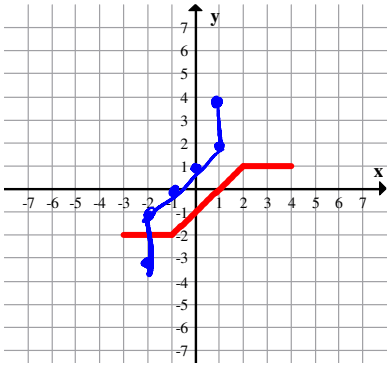


$(0.5,-3)$
$(1.5,-3)$
$(3,0)$
$(4,0)$

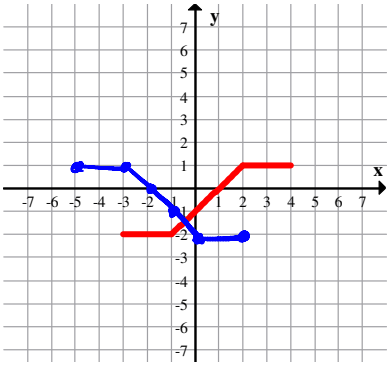
c. $(3x, -y+2)$



d. $f^{-1}(x) \rightarrow (y, x)$



e. $f(-1(x+1)) \rightarrow (-x-1)$



2. Determine the equation of $y = 2x^2 - x + 3$ after a reflection in the:

a. x-axis $y = -(2x^2 - x + 3)$

$y = -2x^2 + x - 3$

b. y-axis $y = 2(-x)^2 - (-x) + 3$

$y = 2x^2 + x + 3$

3. If $(-2,3)$ is on the graph of $y = f(x)$, find a point that must be on:

a. $y = f(-2x-6) - 3$

$y = f(-2(x+3)) - 3$

$(x,y) \to (-\frac{1}{2}x - 3, y - 3)$

$(-2, 0)$

b. $y = -f\left(\frac{1}{2}x - 4\right) + 2$

$y = -f\left(\frac{1}{2}(x-8)\right) + 2$

$\to (2x+8, -y+2)$

$(4, -1)$

c. $y = f^{-1}(x) + 1$

$\to (y, x+1)$

$(3, -1)$

d. $y = f^{-1}(x-1) - 3$

$\to (y+1, x-3)$

$(4, -5)$

4. Find the inverse of the following functions:

a. $f(x) = \frac{1}{2}x + 1$

$x = \frac{1}{2}y + 1$

$2(x-1) = y$

b. $f(x) = \frac{2}{2x+3}$

$x = \frac{2}{2y+3}$

$x(2y+3) = 2$

$2y + 3 = \frac{2}{x}$

$2y = \frac{2}{x} - 3$

$y = \frac{2}{2x} - \frac{3}{2}$

$y = \frac{1}{x} - \frac{3}{2}$

5. Given $f(x) = 2x^2 + x + 3$ and $g(x) = 3x + 1$, find:

a. $f(x) + g(x)$

$= 2x^2 + 4x + 4$

b. $(f - g)(x)$

$= 2x^2 - 2x + 2$

c. $g(f(1))$

$= 19$

d. $f(g(1))$

$= 39$

e. $g(f(-3))$

$= 55$

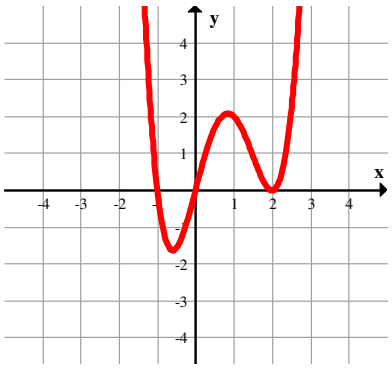
f. $(g \circ f)(x)$

$= 3(2x^2 + x + 3) + 1$

$= 6x^2 + 3x + 10$

Ch 2 Polynomials

1. Sketch and determine the domain of: $y = x(x-2)^2(x+1)$.



$x \in R$

2. Find the equation of the function with zeroes of $\frac{3}{2}, 4,$ and -1 and passing through $(2,3)$.

$y = -\frac{1}{2}(2x-3)(x-4)(x+1)$

3. Factor fully: $f(x) = x^3 - 4x^2 - 7x + 10$ $= (x-1)(x-5)(x+2)$

4. Solve by factoring:

a. $2x^3 - 5x^2 - x + 6 = 0$ $x = -1, \frac{3}{2}, 2$

b. $2x^3 - 5x^2 - 11x = 4$ $x = -1, -\frac{1}{2}, 4$

c. $2x^3 + 7x^2 + 2x = 3$ $x = -1, \frac{1}{2}, -3$

5. Solve:

a. $-x(x+3)(2x-5) \geq 0$ $x \leq -3, \ 0 \leq x \leq \frac{5}{2}$

b. $x^4 - 9x^2 \leq 0$ $x^2(x^2-9) \leq 0$ $x^2(x-3)(x+3) \leq 0$ $-3 \leq x \leq 3$

6. Find the remainder when $3x^3 + 4x^2 - x + 2$ is divided by $x+2$.

$f(-2) = -4$

7. When $x^3 + kx^2 - 6x + 4$ is divided by $x-4$, the remainder is -36. Find the value of k.

$k = -5$

8. Divide the following:

a. $\dfrac{3x^3 - x^2 + 2x + 4}{x+4}$ $= 3x^2 - 13x + 54 - \frac{212}{x+4}$

b. $\left(x^4 - x^2 + 7\right) \div (x+1)$ $= x^3 - x^2 + \frac{7}{x+1}$

9. A piece of cardstock 40cm long and 10cm wide is used to make an open top box by cutting a square from each corner. What is the length of square that must be cut from each corner if the volume of the box must be 408 cm^3. $V = x(40-2x)(10-2x) = 408$

$x = 3$ cm $\quad x = 1.67$ cm

Ch 3 Radical and Rational Functions

1. Find the domain and range for each:

a. $y = \sqrt{3-x}$ $x \leq 3$; $y \geq 0$

b. $y = -\sqrt{2x+7} - 1$ $x \geq -\frac{7}{2}$; $y \leq -1$

c. $f(x) = -\sqrt{-x} - 3$ $x \leq 0$; $y \leq -3$

2. Solve:

a. $2\sqrt{x-3} + 5 = 15$ $x > 3$

$\sqrt{x-3} = 5$

$x - 3 = 25$

$x = 28$ ✓

b. $\sqrt{x} + \sqrt{x-16} = 8$ $x \geq 16$

$\left(\sqrt{x-16}\right)^2 = (8-\sqrt{x})^2$

$x - 16 = 64 - 16\sqrt{x} + x$

$-80 = -16\sqrt{x}$

$5 = \sqrt{x}$

$25 = x$ ✓

c. $2x = \sqrt{x+3} - 5$ $x > 5/2$

$(2x-5)^2 = \left(\sqrt{x+3}\right)^2$

$4x^2 - 20x + 25 = x + 3$

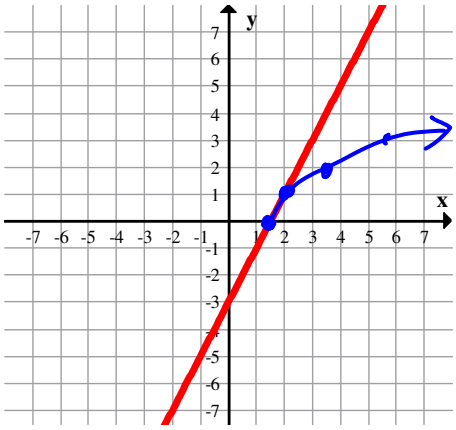
$4x^2 - 21x + 22 = 0$

$x = \dfrac{21 \pm \sqrt{21^2 - 4(4)(22)}}{2(4)}$

$x = 3.804$, ~~1.446~~

3. Given $y = f(x)$, sketch $y = \sqrt{f(x)}$:

a.



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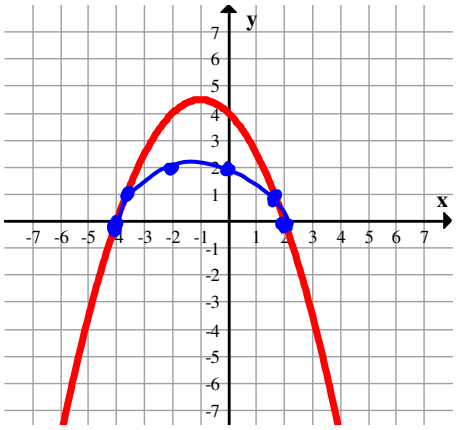


4. Given the following functions, find the equations of all asymptotes, the coordinates of any holes, and the x and y-intercepts:

a. $f(x) = \frac{1}{4} - \frac{1}{x-4}$

$f(x) = \frac{x-8}{4(x-4)}$

ha: $y = \frac{1}{4}$

va: $x = 4$

x-int: $(8,0)$

y-int: $(0, \frac{1}{2})$

b. $f(x) = \frac{2}{x+1} + \frac{3}{x}$

$f(x) = \frac{5x+3}{x(x+1)}$

ha: $y = 0$

va: $x = 0, x = -1$

x-int: none

y-int: none

c. $y = \frac{3x-2}{x+1}$

ha: none

va: $x = -1$

x-int: $(\frac{2}{3}, 0)$

y-int: $(0, -2)$

d. $y = \frac{x^2+4x}{x^2+9x+20} = \frac{x(x+4)}{(x+4)(x+5)}$

ha: $y = 1$

va: $x = -5$

hole: $(-4, -4)$

x-int: $(0,0)$

y-int: $(0,0)$

e. $y = \frac{x^2+6x+8}{x^2-2x-8} = \frac{(x+4)(x+2)}{(x-4)(x+2)}$

ha: $y = 1$

va: $x = 4$

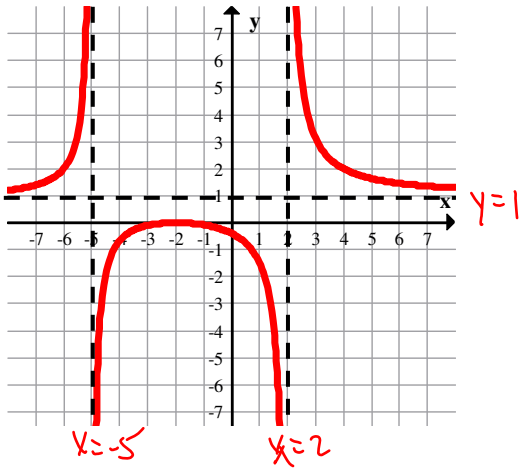
hole: $(-2, \frac{1}{3})$

x-int: $(-4, 0)$

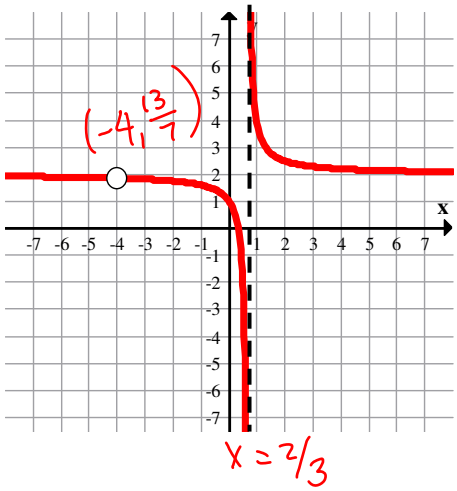
y-int: $(0, -1)$

5. Sketch the following rational expressions. Label holes and asymptotes:

a. $y = \frac{x^2+4x+4}{x^2+3x-10} = \frac{(x+2)^2}{(x+5)(x-2)}$



b. $f(x) = \frac{2(3x-1)(x+4)}{3x^2+10x-8} = \frac{2(3x-1)(x+4)}{(3x-2)(x+4)}$



hole: $x = -4$

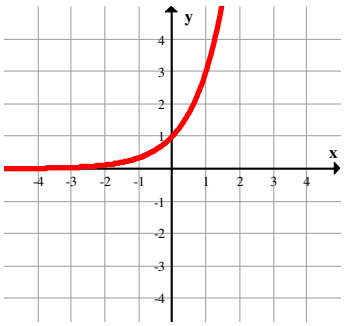
$y = \frac{2(-12-1)}{(-12)-2}$

$= \frac{+26}{+14} = \frac{13}{7}$

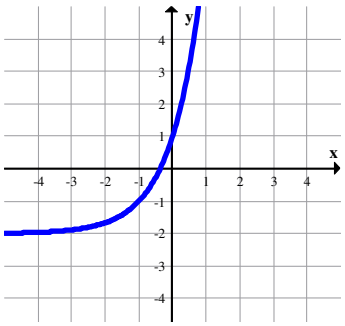
Ch 4 Logarithms

1. Graph $y = 3^x$ and $y = 3^{x+1} - 2$. Find the domain, range and give the equations of any asymptotes.

$y = 3^x$
$x \in \mathbb{R}$
$y > 0$
$y = 0$



$y = 3^{x+1} - 2$
$x \in \mathbb{R}$
$y > -2$
$y = -2$



2. Solve for x:

a. $\left(\frac{1}{81}\right)^{3x-2} = 27^{2x-1}$

3^{-4} 3^3

$-12x + 8 = 6x - 3$
$-18x = -11$
$x = \frac{11}{18}$

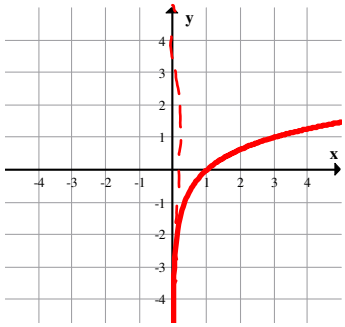
b. $2^{x^2} = \left(16^{x-1}\right)\left(2^x\right)$

$2^{4(x-1)} + x$

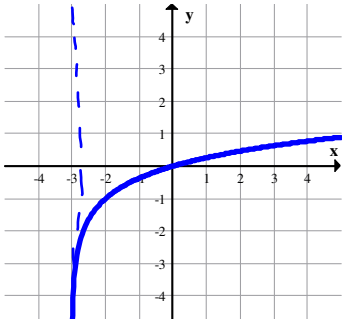
$x^2 = 5x - 4$
$x^2 - 5x + 4 = 0$
$(x-4)(x-1) = 0$
$x = 1, \; x = 4$

3. Graph $y = \log_3 x$ and $y = \log_3(x+3) - 1$. Find the domain, range and give the equations of any asymptotes.

$y = \log_2 x$
$x > 0$
$y \in \mathbb{R}$
$x = 0$



$x > -3$
$y \in \mathbb{R}$
$x = -3$



4. Find the value of x:

a. $\log_{125} x = \frac{2}{3}$

$x = 25$

b. $\log_9 \frac{1}{81} = x$

$x = -2$

c. $\log_9 \frac{1}{27} = x$

$3^{2x} = 3^{-3}$ $\therefore x = -\frac{3}{2}$

d. $\log_x 8 = \frac{3}{4}$

$\left(x^{3/4}\right)^{4/3} = (8)^{4/3}$
$x = 16$

e. $6^{\log x} = \frac{1}{36}$

$6^{\log x} = 6^{-2}$
$x = 10^{-2} = \frac{1}{100}$

f. $\log_4 8^x = 8$

$4^8 = 8^x$
$2^{16} = 2^{3x}$
$\frac{16}{3} = x$

5. Solve:

a. $2\log m^2 + 3\log m^3 = 10$

$\log m^5 = 10$
$10^{10} = m^5$
$m = 10^2 = 100$

b. $\log_2(2m+4) - \log_2(m-1) = 3$

$\log_2\left(\frac{2m+4}{m-1}\right) = 3$
$8m - 8 = 2m + 4$
$m = 2$

6. The point $(1024, 5)$ (x, y) goes through the function $y = \log_a x$. What is a?

$5 = \log_a 1024$

$a^5 = 1024$
$a = 4$

7. Solve to three decimal places: $8^{5x-2} = 69$.

$(5x-2)\log 8 = \log 69$

$5x\log 8 - 2\log 8 = \log 69$

$x = \dfrac{\log 69 + 2\log 8}{5\log 8} = 0.807$

8. The half-life of a substance is 23 days. How long will it be until the amount remaining is 10% of the initial amount?

$0.1 = \left(\frac{1}{2}\right)^{t/23}$

$\log 0.1 = \frac{t}{23}\log 0.5$

$76.4 \text{ day} \approx t$

9. Two earthquakes measure 7.7 and 6.3 on the Richter scale. How many times more powerful is the first than the second?

$\dfrac{10^{7.7}}{10^{6.3}} = 25.12$ times more powerful

10. Simplify: $3^{\log_3(2x+1)^2}$.

$(2x+1)^2 = 4x^2 + 4x + 1$

11. If $\log_3 x = 2$ and $\log_3 y = 5$, find:

a. $\log_3\left(9x^2y\right)$

$= \log_3 9 + 2\log_3 x + \log_3 y$

$= 2 + 2(2) + 5$

$= 11$

b. $\log_9\left(\dfrac{3x^2}{y}\right)$

$= \log_9 3 + 2\log_9 x - \log_9 y$

$= \frac{1}{2} + 2\log_3 x^{\frac{1}{2}} - \log_3 y^{\frac{1}{2}}$

$= \frac{1}{2} + \log_3 x - \frac{1}{2}\log_3 y$

$= \frac{1}{2} + 2 - \frac{1}{2}\times 5$

$= 0$